Exercise 9A

1 a $\frac{dy}{dx} = x^5$

$y = \frac{x^6}{6} + c$

b $\frac{dy}{dx} = 10x^4$

$y = \frac{10x^5}{5} + c$

$y = 2x^5 + c$

c $\frac{dy}{dx} = -x^{-2}$

$y = -\frac{x^{-1}}{-1} + c$

$y = x^{-1} + c$ or

$y = \frac{1}{x} + c$

d $\frac{dy}{dx} = -4x^{-3}$

$y = \frac{-4x^{-2}}{-2} + c$

$y = 2x^{-2} + c$ or

$y = \frac{2}{x^2} + c$

e $\frac{dy}{dx} = x^{\frac{2}{3}}$

$y = \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + c$

$y = \frac{3}{5}x^{\frac{5}{3}} + c$

f $\frac{dy}{dx} = 4x^{\frac{1}{2}}$

$y = 4\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$y = \frac{8}{3}x^{\frac{3}{2}} + c$

1 g $\frac{dy}{dx} = -2x^6$

$y = -\frac{2}{7}x^7 + c$

h $\frac{dy}{dx} = x^{-\frac{1}{2}}$

$y = \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$y = 2x^{\frac{1}{2}} + c$ or

$y = 2\sqrt{x} + c$

i $\frac{dy}{dx} = 5x^{-\frac{3}{2}}$

$y = \frac{5x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$

$y = -10x^{-\frac{1}{2}} + c$ or

$y = -\frac{10}{\sqrt{x}} + c$

j $\frac{dy}{dx} = 6x^{\frac{1}{3}}$

$y = \frac{6x^{\frac{4}{3}}}{\frac{4}{3}} + c$

$y = \frac{18}{4}x^{\frac{4}{3}} + c$

$y = \frac{9}{2}x^{\frac{4}{3}} + c$

k $\frac{dy}{dx} = 36x^{11}$

$y = \frac{36x^{12}}{12} + c$

$y = 3x^{12} + c$

1 l $\frac{dy}{dx} = -14x^{-8}$

$y = \frac{-14x^{-7}}{-7} + c$

$y = 2x^{-7} + c$ or

$y = \frac{2}{x^7} + c$

m $\frac{dy}{dx} = -3x^{-\frac{2}{3}}$

$y = \frac{-3x^{\frac{1}{3}}}{\frac{1}{3}} + c$

$y = -9x^{\frac{1}{3}} + c$

n $\frac{dy}{dx} = -5$

$= -5x^0$

$y = \frac{-5x^1}{1} + c$

$y = -5x + c$

o $\frac{dy}{dx} = 6x$

$y = \frac{6x^2}{2} + c$

$y = 3x^2 + c$

p $\frac{dy}{dx} = 2x^{-0.4}$

$y = \frac{2x^{0.6}}{0.6} + c$

$y = \frac{20}{6}x^{0.6} + c$

$y = \frac{10}{3}x^{0.6} + c$

2 a $\frac{dy}{dx} = x^3 - \frac{3}{2}x^{-\frac{1}{2}} - 6x^{-2}$

$y = \frac{x^4}{4} - \frac{3}{2}\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - 6\frac{x^{-1}}{-1} + c$

$y = \frac{1}{4}x^4 - 3x^{\frac{1}{2}} + 6x^{-1} + c$

2 b $\frac{dy}{dx} = 4x^3 + x^{-\frac{2}{3}} - x^{-2}$

$y = \frac{4x^4}{4} + \frac{x^{\frac{1}{3}}}{\frac{1}{3}} - \frac{x^{-1}}{-1} + c$

$y = x^4 + 3x^{\frac{1}{3}} + x^{-1} + c$

c $\frac{dy}{dx} = 4 - 12x^{-4} + 2x^{-\frac{1}{2}}$

$y = 4x - \frac{12x^{-3}}{-3} + \frac{2x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$y = 4x + 4x^{-3} + 4x^{\frac{1}{2}} + c$

d $\frac{dy}{dx} = 5x^{\frac{2}{3}} - 10x^4 + x^{-3}$

$y = \frac{5x^{\frac{5}{3}}}{\frac{5}{3}} - \frac{10x^5}{5} + \frac{x^{-2}}{-2} + c$

$y = 3x^{\frac{5}{3}} - 2x^5 - \frac{1}{2}x^{-2} + c$

e $\frac{dy}{dx} = -\frac{4}{3}x^{-\frac{4}{3}} - 3 + 8x$

$y = -\frac{4}{3}\frac{x^{-\frac{1}{3}}}{-\frac{1}{3}} - 3x + \frac{8x^2}{2} + c$

$y = 4x^{-\frac{1}{3}} - 3x + 4x^2 + c$

f $\frac{dy}{dx} = 5x^4 - x^{-\frac{3}{2}} - 12x^{-5}$

$y = \frac{5x^5}{5} - \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} - \frac{12x^{-4}}{-4} + c$

$y = x^5 + 2x^{-\frac{1}{2}} + 3x^{-4} + c$

3 a $f'(x) = 12x + \frac{3}{2}x^{-\frac{3}{2}} + 5$

$f(x) = \frac{12x^2}{2} + \frac{3}{2}\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + 5x + c$

$f(x) = 6x^2 - 3x^{-\frac{1}{2}} + 5x + c$

3 b $f'(x) = 6x^5 + 6x^{-7} - \frac{1}{6}x^{-\frac{7}{6}}$

$$f(x) = \frac{6x^6}{6} + \frac{6x^{-6}}{-6} - \frac{1}{6}\frac{x^{-\frac{1}{6}}}{-\frac{1}{6}} + c$$

$$f(x) = x^6 - x^{-6} + x^{-\frac{1}{6}} + c$$

c $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$

$$f(x) = \frac{1}{2}\frac{x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2}\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + c$$

$$f(x) = x^{\frac{1}{2}} + x^{-\frac{1}{2}} + c$$

d $f'(x) = 10x^4 + 8x^{-3}$

$$f(x) = \frac{10x^5}{5} + \frac{8x^{-2}}{-2} + c$$

$$f(x) = 2x^5 - 4x^{-2} + c$$

e $f'(x) = 2x^{-\frac{1}{3}} + 4x^{-\frac{5}{3}}$

$$f(x) = \frac{2x^{\frac{2}{3}}}{\frac{2}{3}} + \frac{4x^{-\frac{2}{3}}}{-\frac{2}{3}} + c$$

$$f(x) = 3x^{\frac{2}{3}} - 6x^{-\frac{2}{3}} + c$$

f $f'(x) = 9x^2 + 4x^{-3} + \frac{1}{4}x^{-\frac{1}{2}}$

$$f(x) = \frac{9x^3}{3} + \frac{4x^{-2}}{-2} + \frac{1}{4}\frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$$

$$f(x) = 3x^3 - 2x^{-2} + \frac{1}{2}x^{\frac{1}{2}} + c$$

4 $$\frac{dy}{dx} = (2x+3)^2$$

$$= 4x^2 + 12x + 9$$

$$y = \frac{4}{3}x^3 + 6x^2 + 9x + c$$

5 $f'(x) = 3x^{-2} + 6x^{\frac{1}{2}} + x - 4$

$$f(x) = \frac{3x^{-1}}{-1} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^2}{2} - 4x + c$$

$$= -3x^{-1} + 4x^{\frac{3}{2}} + \frac{1}{2}x^2 - 4x + c$$

Challenge

$$\frac{dy}{dx} = (2\sqrt{x} - x^2)\left(\frac{3+x}{x^5}\right)$$

$$= (2x^{\frac{1}{2}} - x^2)(3x^{-5} + x^{-4})$$

$$= 6x^{-\frac{9}{2}} + 2x^{-\frac{7}{2}} - 3x^{-3} - x^{-2}$$

$$y = \frac{6x^{-\frac{7}{2}}}{-\frac{7}{2}} + \frac{2x^{-\frac{5}{2}}}{-\frac{5}{2}} - \frac{3x^{-2}}{-2} - \frac{x^{-1}}{-1} + c$$

$$= -\frac{12}{7}x^{-\frac{7}{2}} - \frac{4}{5}x^{-\frac{5}{2}} + \frac{3}{2}x^{-2} + \frac{1}{x} + c$$